MVE055/MSG810, Matematisk statistik och diskret matematik, 2016/17
Assignment 2
Due October 3, 2016

1. **Exponential distribution**

   (a) Assume that X has exponential distribution with mean $1/\lambda$. Compute $P(X > x)$.

   (b) Assume that X_1, X_2 are independent exponential random variables with mean $1/\lambda$. Find the distribution function of $min(X_1, X_2)$.

   (c) Assume that there are 3 different brands of light bulbs whose lifetimes are exponential random variables with mean 1/2, 1/3, and 1/5 years, respectively. Assuming that all of the bulbs are independent, what is the expected time before one of the bulbs fail.

2. Consider a Markov chain X_k with state space $\{1, 2, ..., n\}$. Assume that whenever the state is i, a reward $r(i)$ is obtained. Then, $R_k = r(X_0)+r(X_1)+ ... + r(X_k)$ is the total reward obtained over the time interval $\{1, 2, ..., k\}$. For every state i, define:

$$m_k(i) = E[R_k|X_0 = i],$$
$$v_k(i) = Var[R_k|X_0 = i]$$

   (a) Find a recursion formula to compute $m_{k+1}(i)$ for $i \in \{1, 2, ..., n\}$.

   (b) Find a recursion formula to compute $v_{k+1}(i)$ for $i \in \{1, 2, ..., n\}$. (**Hint: Use the law of total variance**)